Post-Quantum Insecurity from LWE

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Does proving security under LWE imply post-quantum security?

For Interactive Protocols — No!

Prior Work: Security proofs for interactive protocols can break down for quantum adversaries

- Zero-Knowledge — [vdG97], [Wat06]
- Computational soundness — [Unr12], [ARU14]

Main Issue: Rewinding

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Main Issue: Rewinding

Not just failure of proof techniques:

- [BCMVV18]: explicit counter example

What about *non-interactive* primitives?

- OWF
- PRG
- PRF
- MAC
- Signatures
- CPA SKE
- CCA PKE
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$$\text{Security from LWE} \overset{?}{\Rightarrow} \text{Post-quantum security}$$

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Security from LWE $\not\Rightarrow$ Post-quantum security

<u>Main Result</u>: explicit (contrived) counterexamples for ***non-interactive*** primitives that are

- Classically secure under LWE
- Quantumly broken

Techniques

Core Observation: Many non-interactive primitives have **interactive security games**

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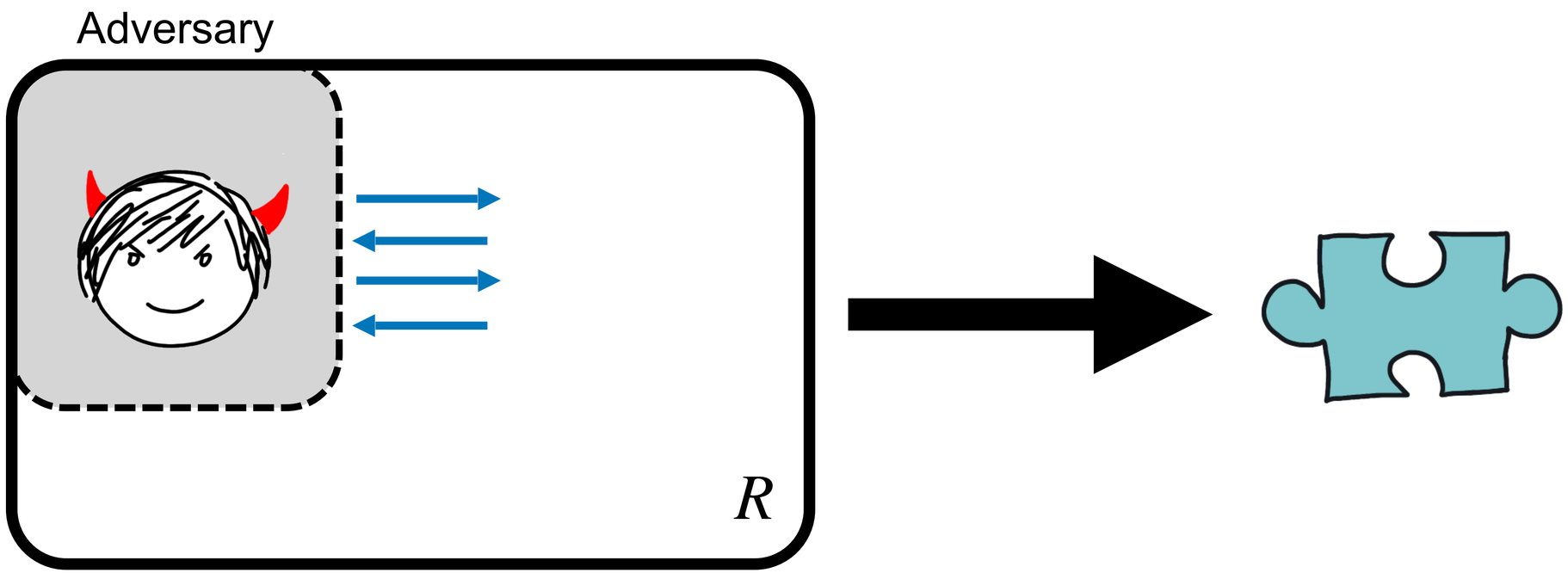
Goal: “Force” the reduction to rewind the adversary

Technique: Embed an “interactive proof of quantumness” into the security game

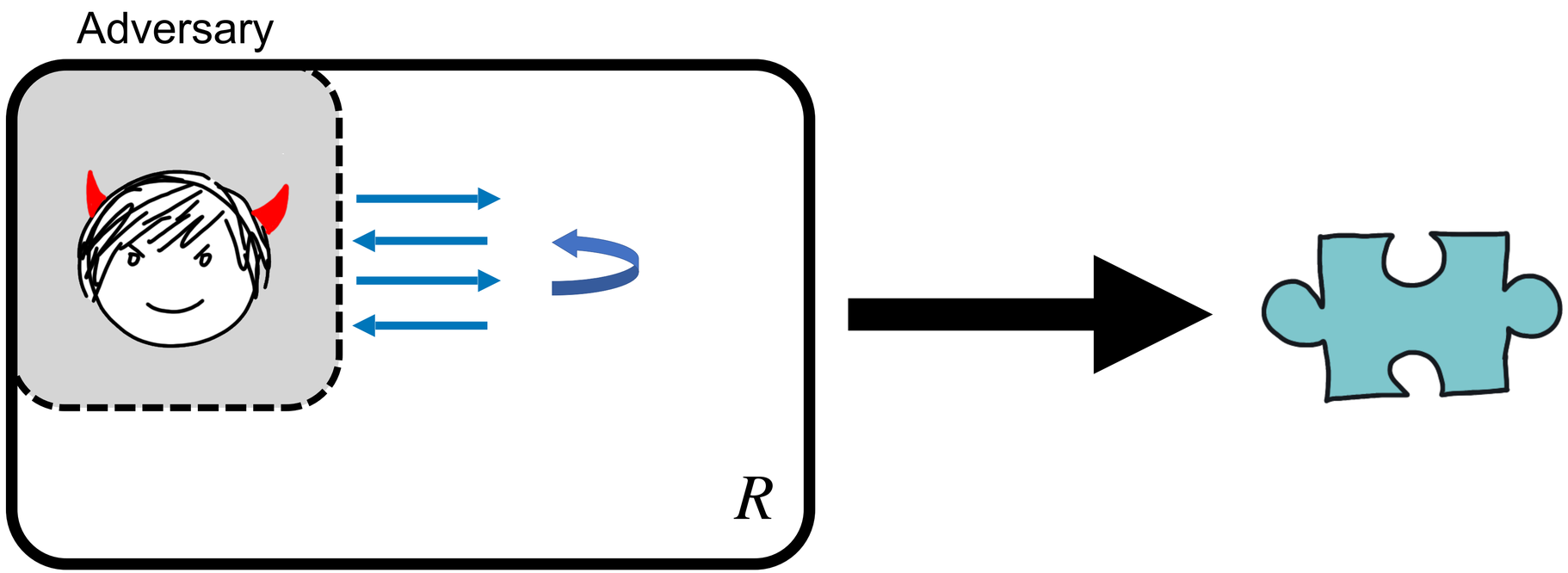
What goes wrong?



Black-box Reduction

Computationally Hard Problem

Rewinding



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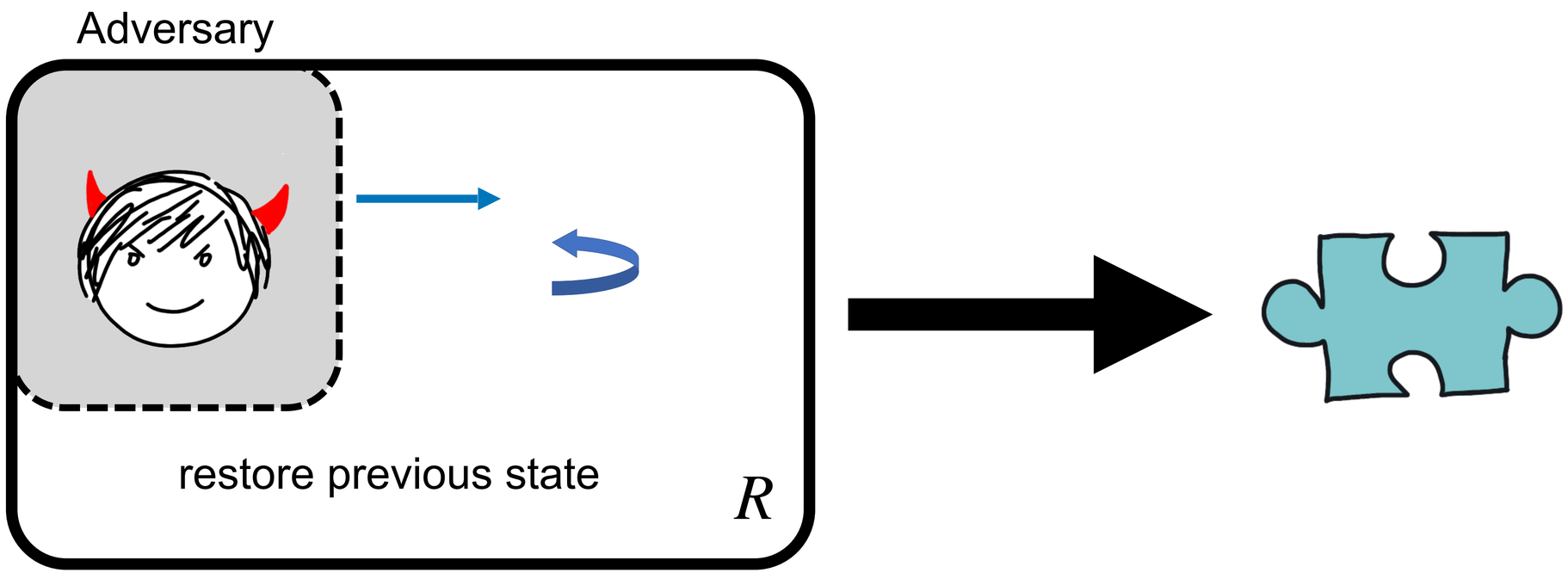

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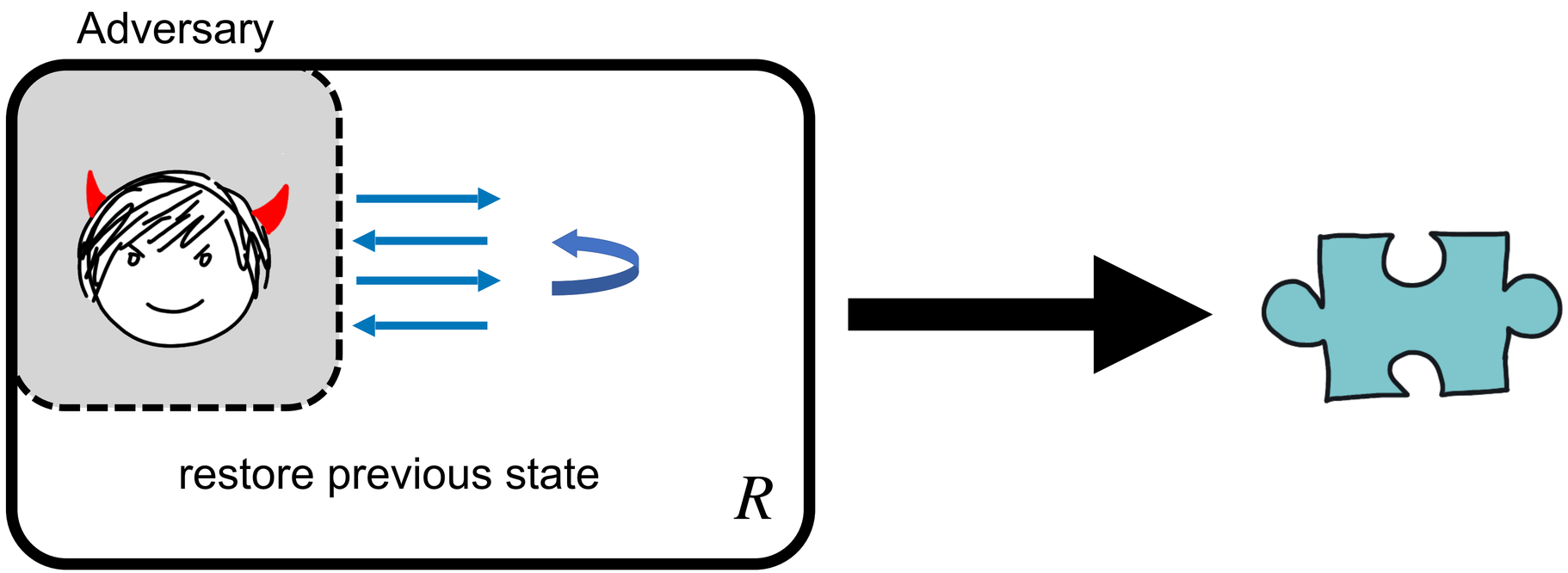

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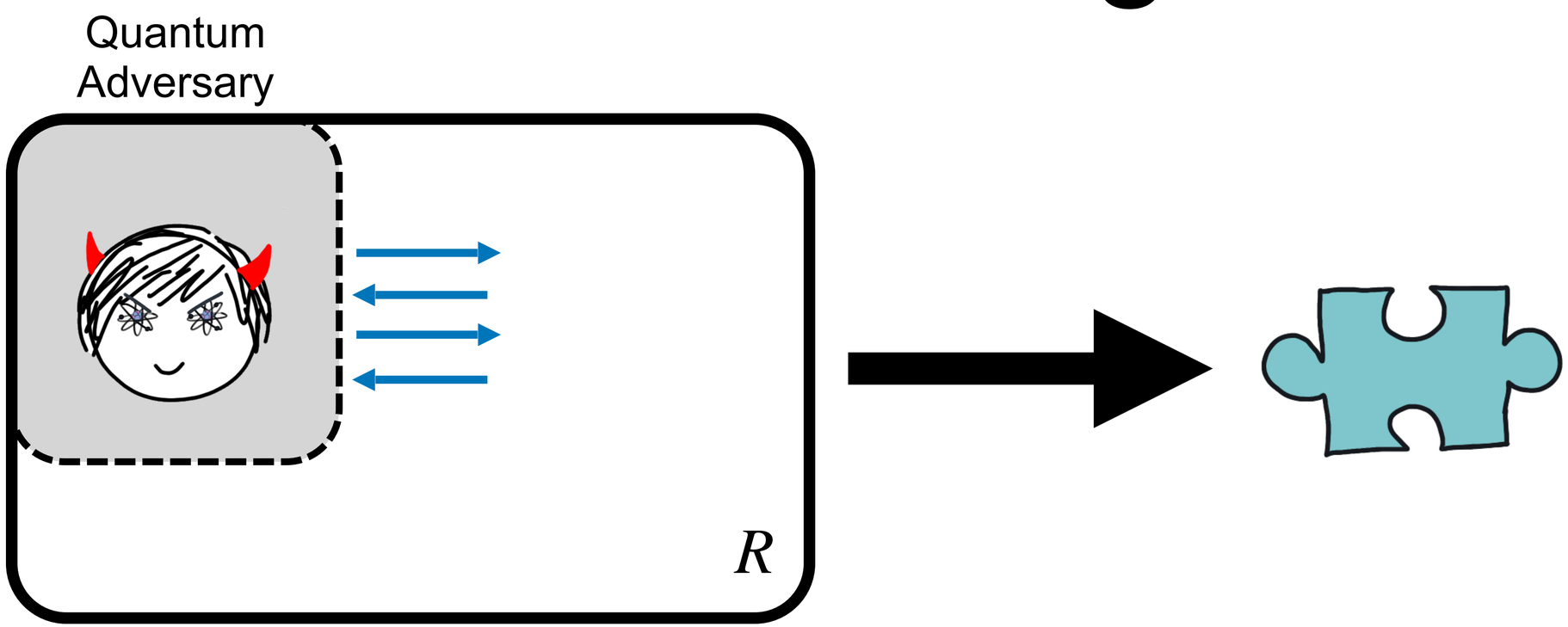

Black-box Reduction

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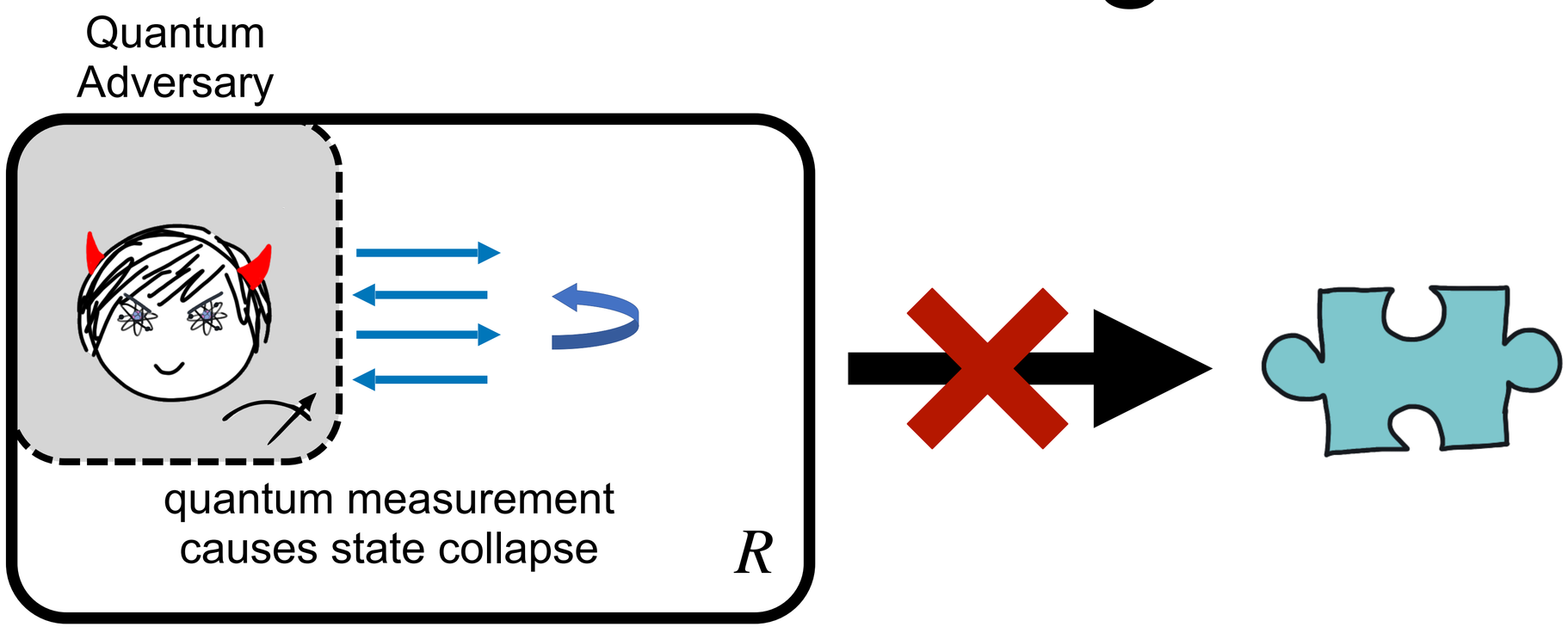

Black-box Reduction

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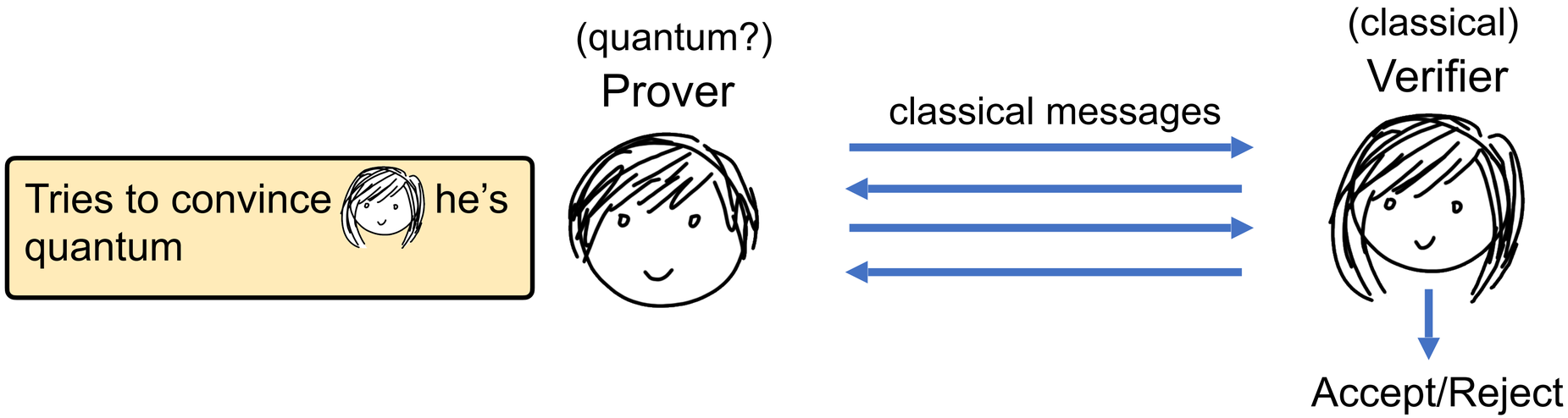
Interactive Proofs of Quantumness (IPQs)

[BCMVV18]



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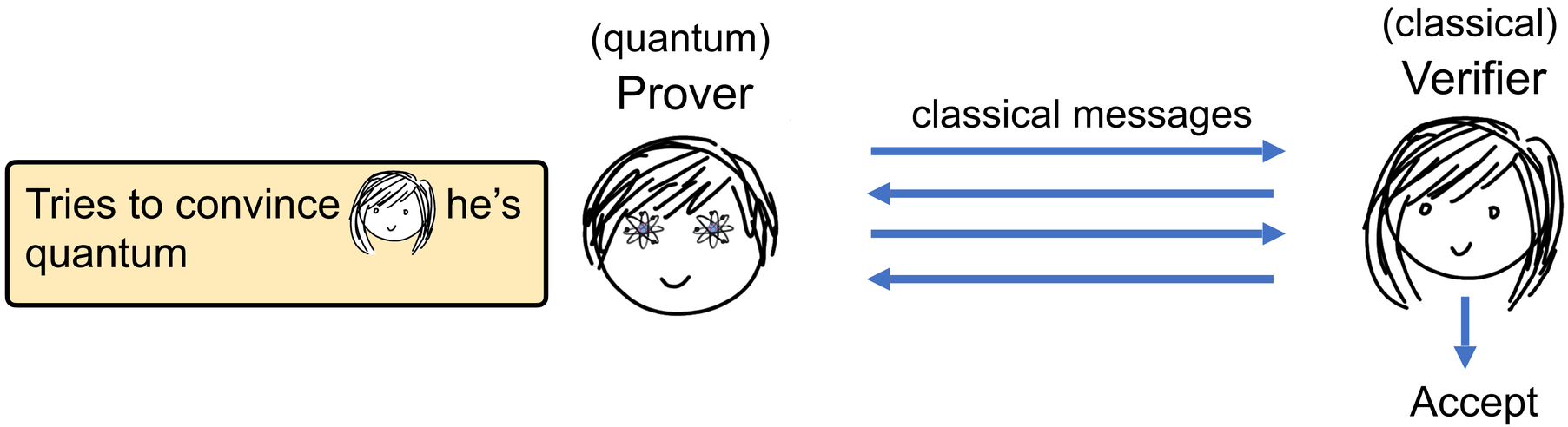

- **Completeness:** quantum can convince

Interactive Proofs of Quantumness (IPQs)

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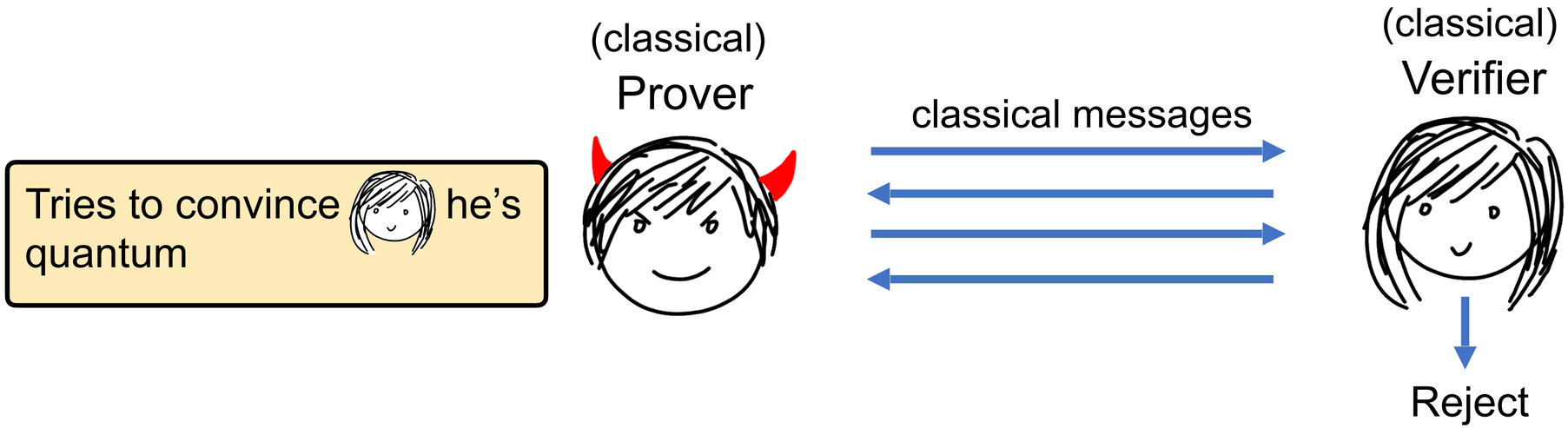
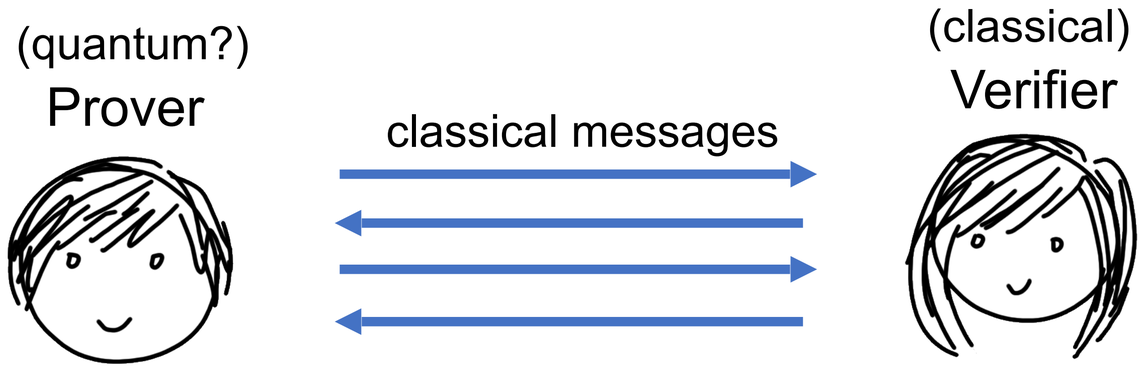


- **Completeness:** quantum can convince
- **Soundness:** classical cannot convince but with negligible probability

Interactive Proofs of Quantumness (IPQs)

[BCMVV18]



IPQs = Primitives where rewinding issues are inherent

Any reduction will fail for quantum adversaries

Interactive Proofs of Quantumness (IPQs)

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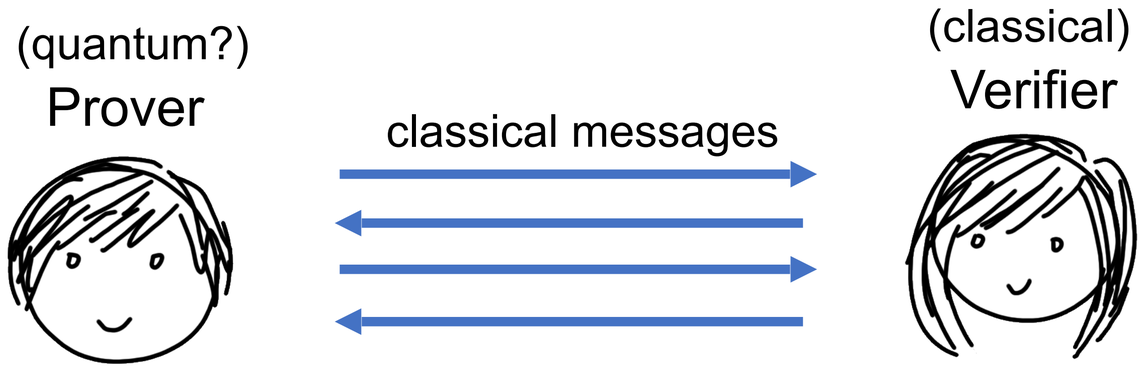

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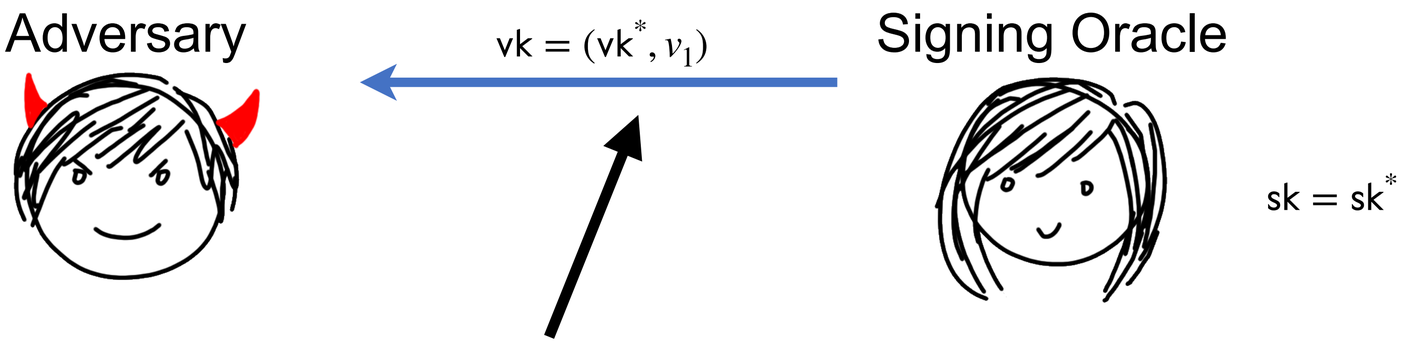
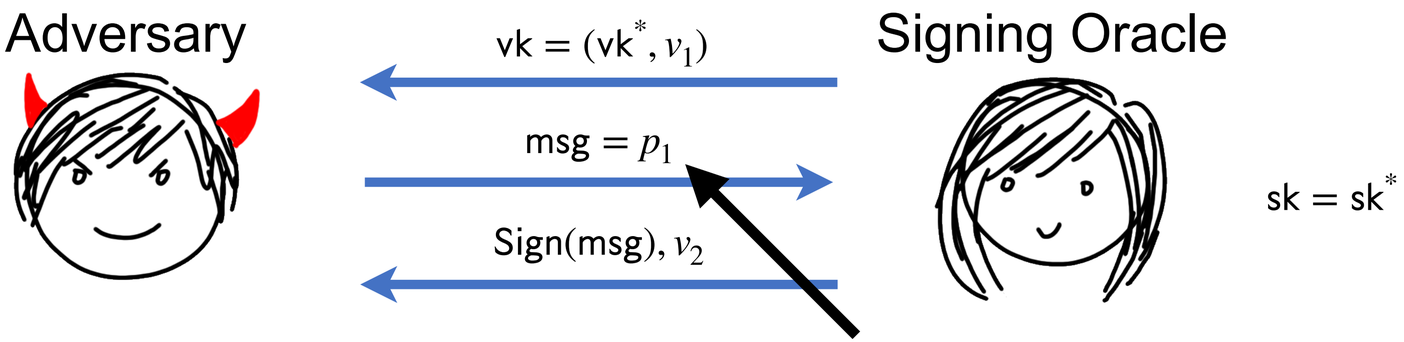
<u>Theorem</u> [BCMVV18]**:** 4-round IPQ from LWE

Embedding an IPQ in Signatures



Verification key contains
first IPQ verifier message

Embedding an IPQ in Signatures



Parse each signing query as an IPQ prover message

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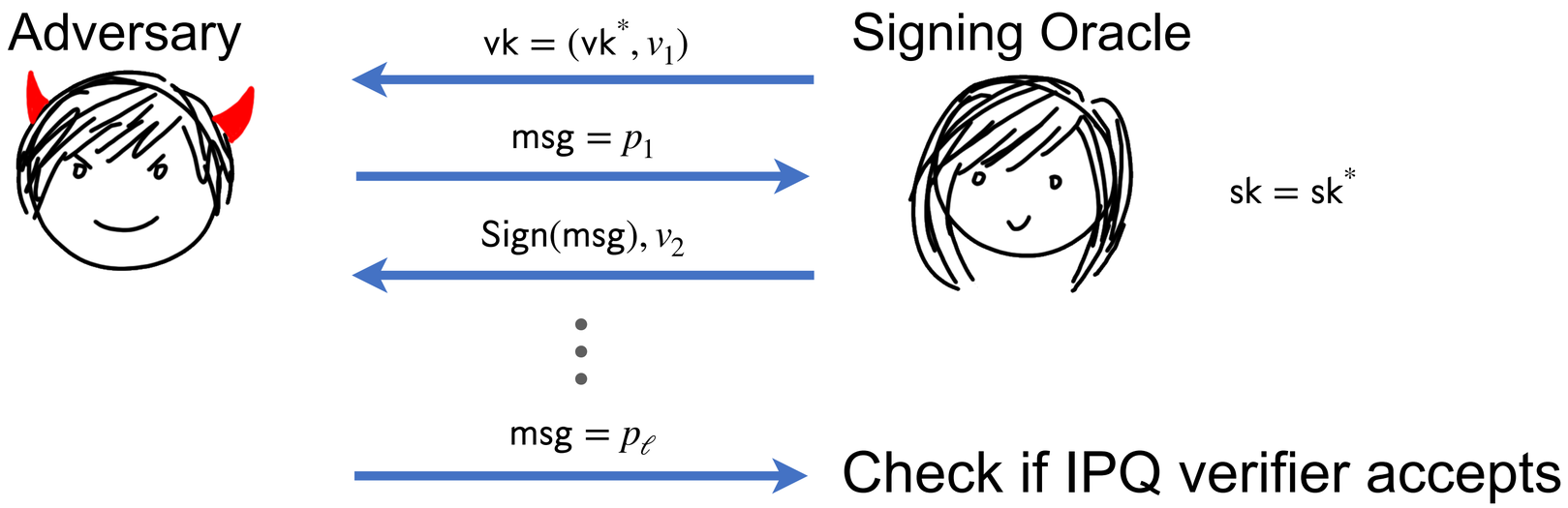

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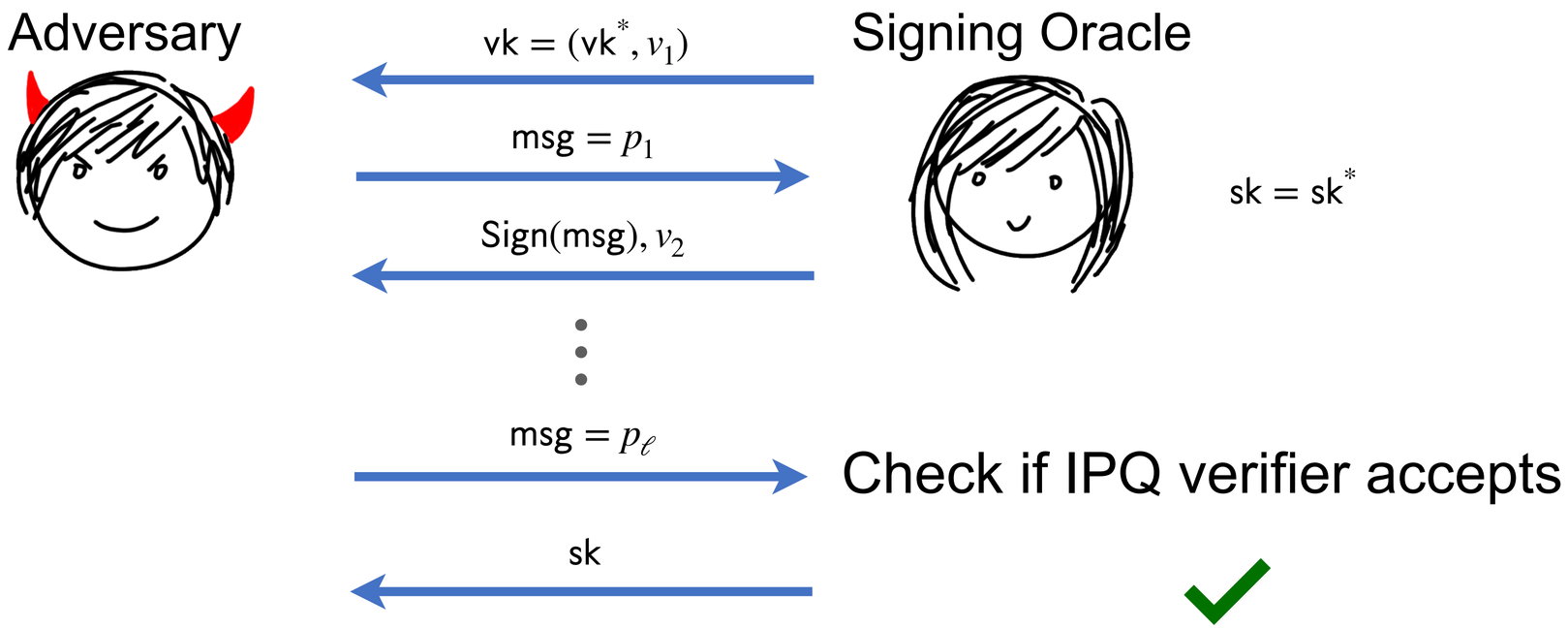

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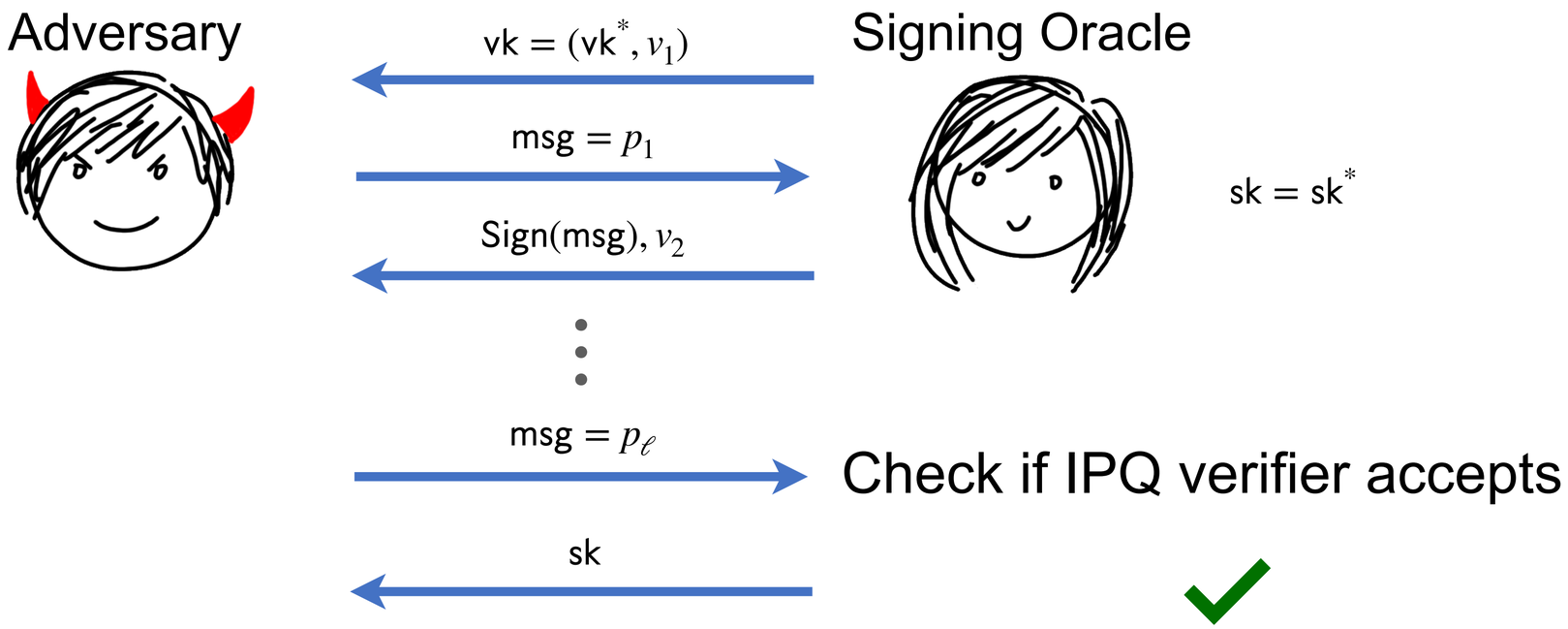
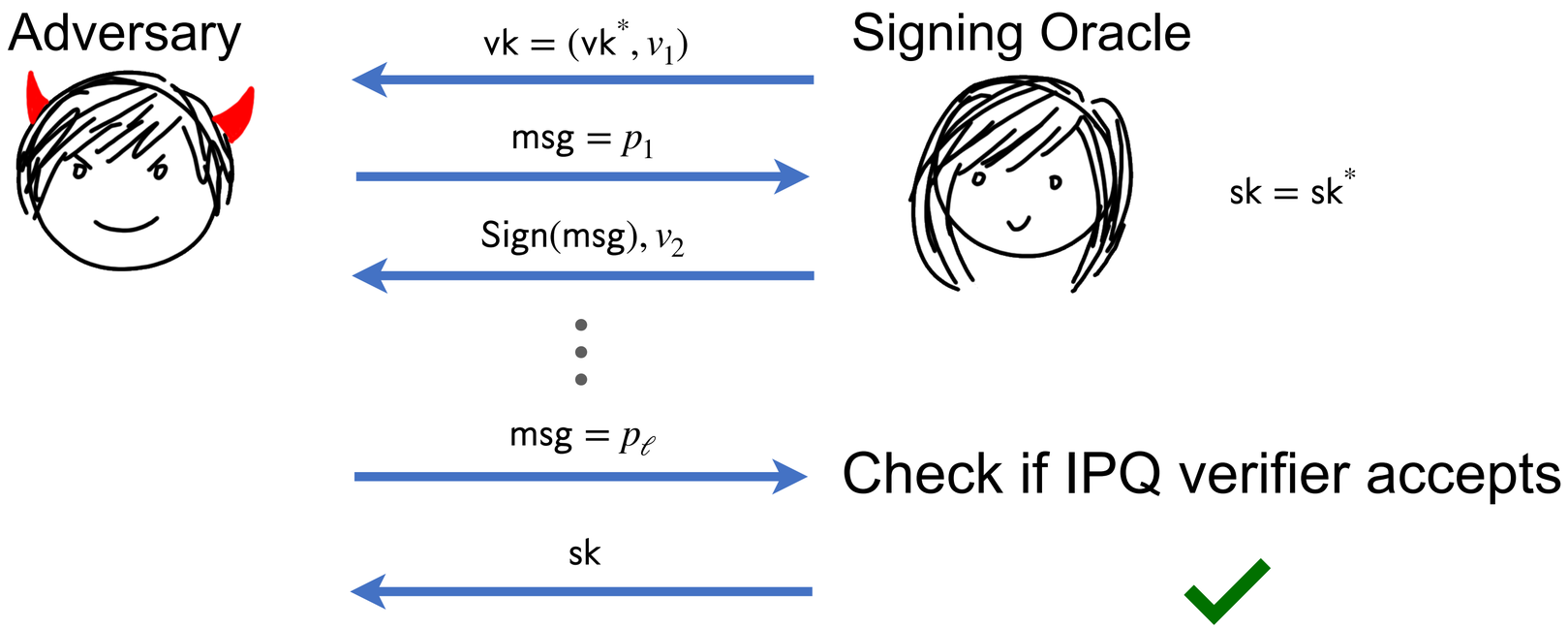


IPQ Soundness ⇒ Classical security

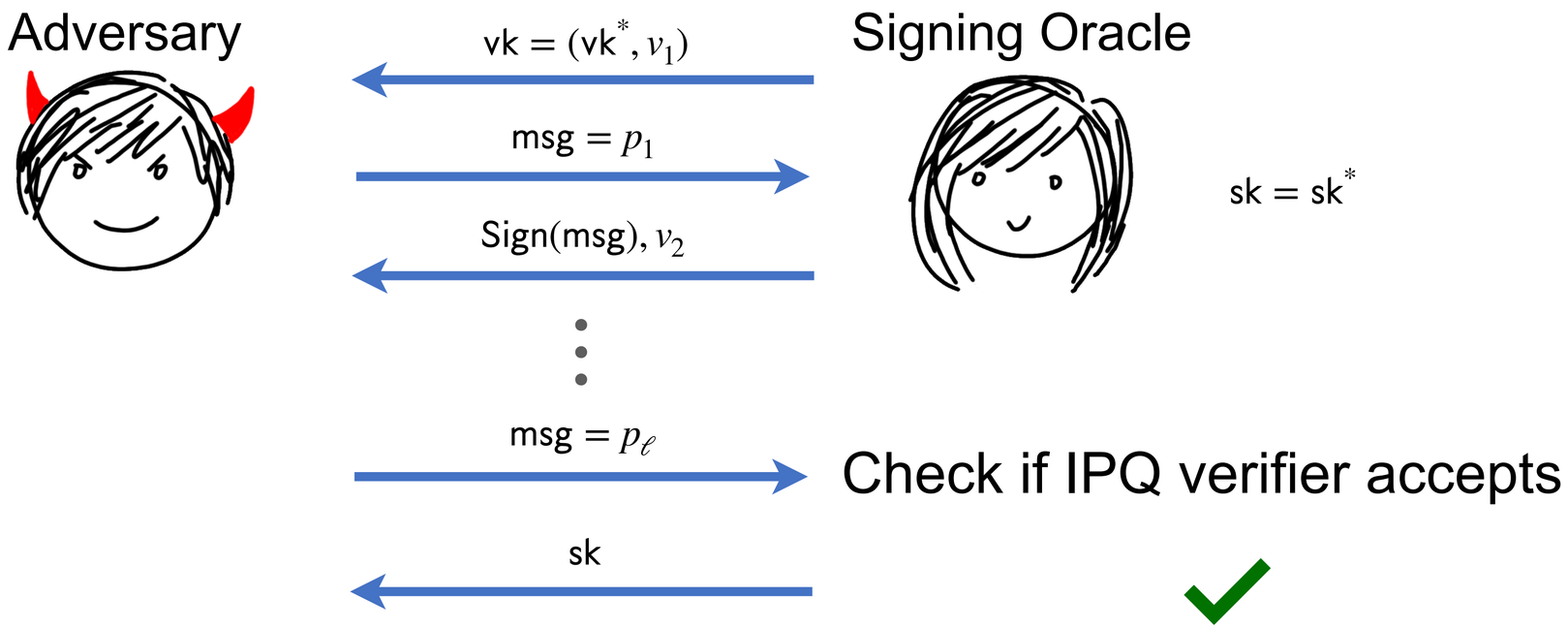
IPQ Completeness ⇒ Quantum break

Embedding an IPQ in Signatures



Problem: Signing oracle now stateful

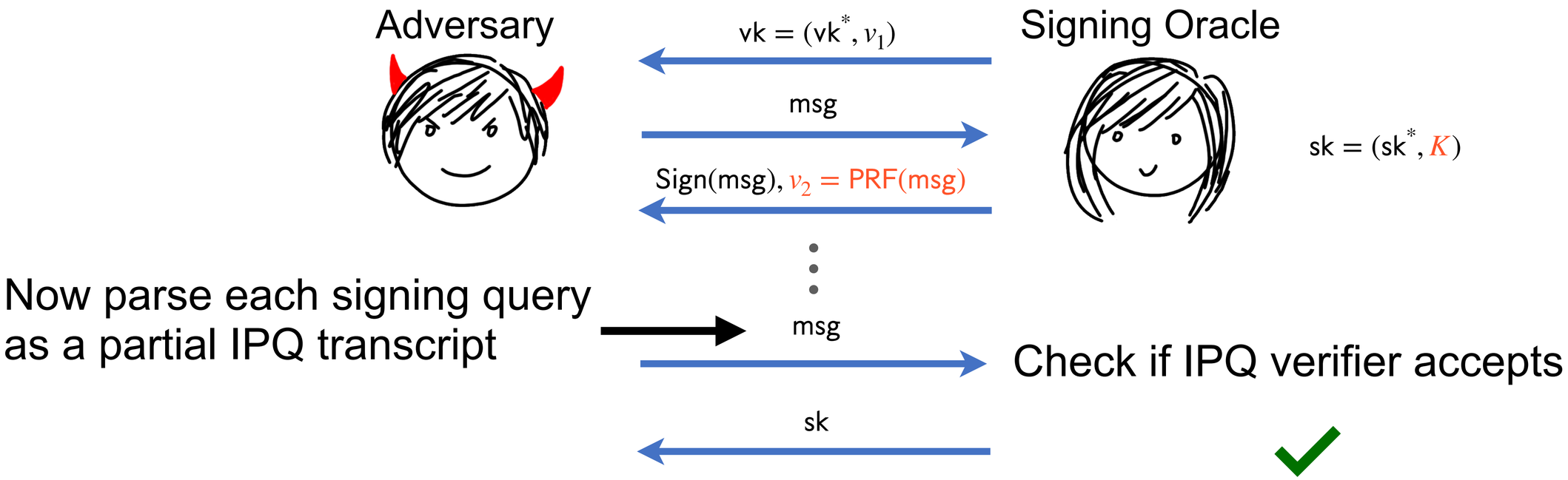
Embedding an IPQ in Signatures



Observation: IPQ from [BCMVV18] is public coin after first verifier message

$\Rightarrow$ it satisfies a notion of *resettable soundness*

Embedding an IPQ in Signatures



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Other Results

<u>Theorem</u>: construct a **3-round** protocol where classical sender sends a message m such that

- m is hidden from classical receivers
- quantum receiver learns m

<u>Theorem</u>: counterexamples for non-interactive primitives that are

- ***"One-time"*** classically secure under LWE
- Quantumly broken in fewer queries

→

- One-time signatures
- One-time MAC
- One-time PRF
- One-time SKE
- One-time CCA PKE

Open Problems

Can we get counterexamples for CPA public-key encryption?

↳ Our techniques fall short b/c adversary win is publicly verifiable

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What about truly non-interactive primitives (OWFs, PRGs,…)?

↳ No interaction in security game ⇒ no rewinding
↳ Seemingly would require non black-box techniques
↳ [YZ22]: Counterexample for OWFs in ROM

Summary

<u>Main Theorem</u>: explicit (contrived) counterexamples for ***non-interactive*** primitives that are

- Classically secure under LWE
- Quantumly broken

- PRF
- Signatures
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<u>Theorem</u>: counterexamples for “one-time” versions of the same primitives

Reductions for post-quantum security must be quantum compatible regardless of “post-quantumness” of assumption